CfE Advanced Higher Physics

Rotational Motion & Astrophysics Past Paper Homework

4. Gravitation

1. The gravitational pull of the Earth keeps a satellite in a circular orbit.

(*a*) Show that for an orbit of radius r the period T is given by

$$T = 2\pi\sqrt{\frac{r^3}{GM_E}}$$

where the symbols have their usual meanings.

(*b*) A polar orbiting satellite is used to map the Earth by photographing strips of the surface as it orbits, as shown in Figure 1

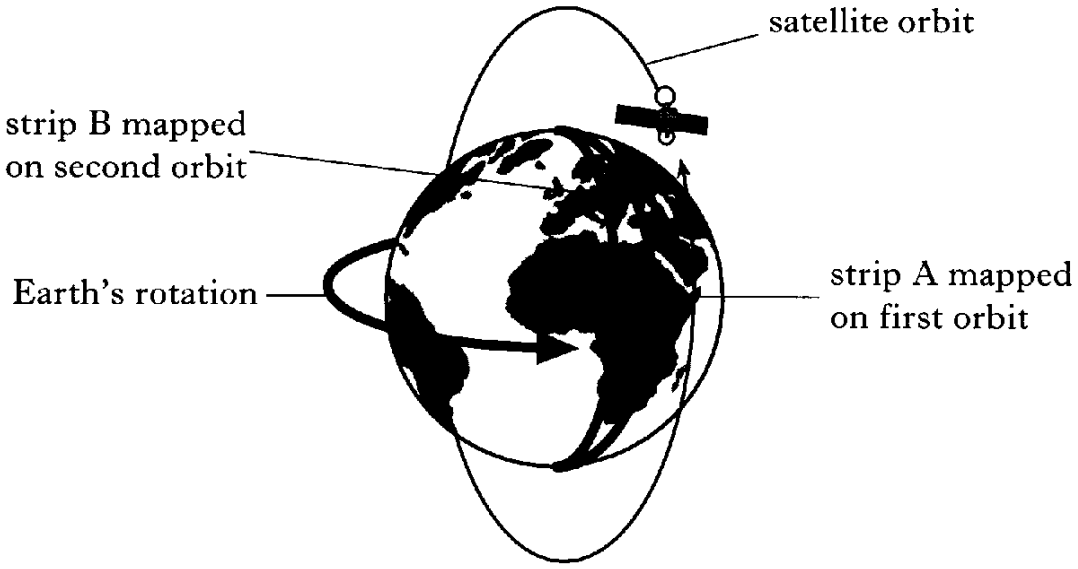

Figure 2

The plane of the satellite orbit is fixed. The Earth rotates and so the satellite maps a different strip on each orbit.

(i) The satellite orbits at a height of 80 km above the surface of the Earth. Assuming the Earth to be spherical, show that the period of the orbit is approximately 86 minutes.

(ii) The Earth's angular velocity is $7{\cdot}3 \times 10^{-5}$ rad s^{-1}.

Calculate the distance along the equator between strips A and B which are mapped on consecutive orbits.

2. (*a*) (i) A satellite orbits a planet of mass M. The orbital radius of the satellite is R and the orbital period is T.
Show that

$$T^2 = \frac{4\pi^2 R^3}{GM}.$$

(ii) Calculate the time taken by the Moon to make one complete orbit of the Earth.

(*b*) A satellite orbits 400 km above the Earth's surface as shown in Figure 2

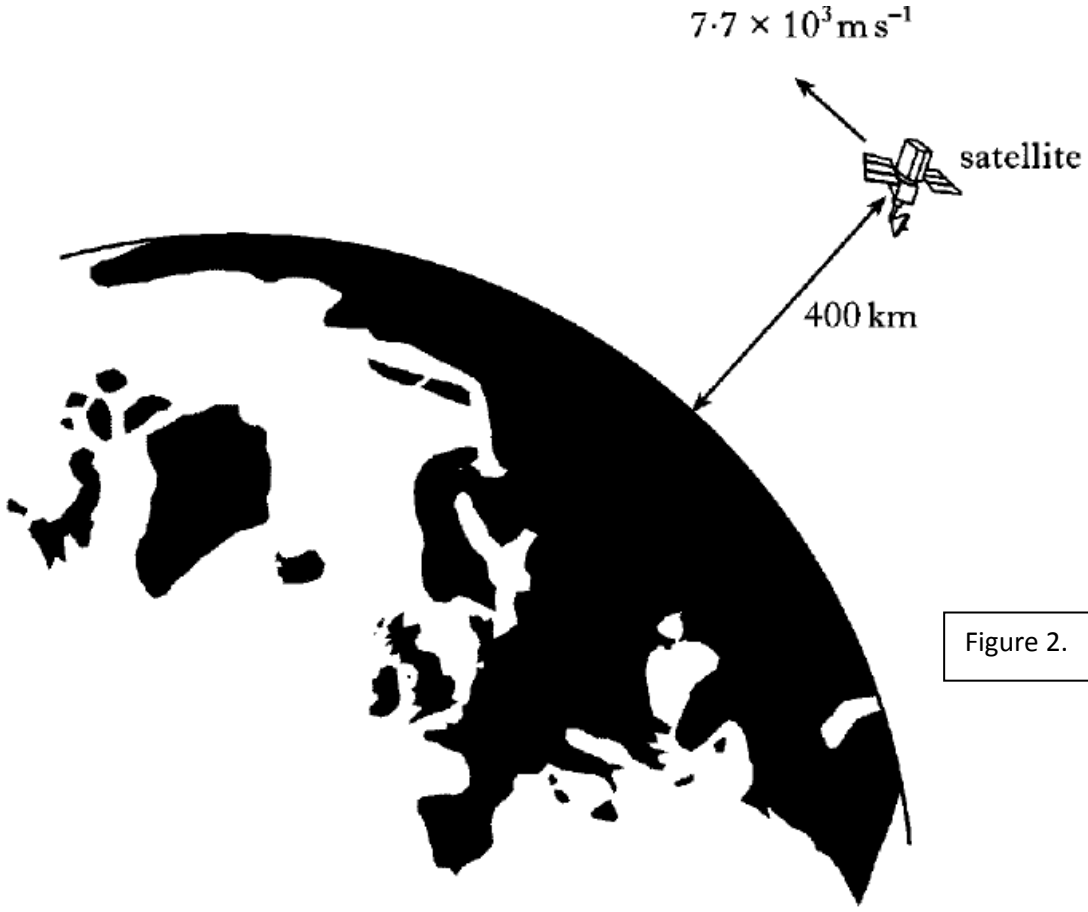

Figure 2.

The satellite has a mass of 900 kg and a speed of $7{\cdot}7 \times 10^3\ \text{m s}^{-1}$.

(i) Show that the potential energy of the satellite is $-5{\cdot}3 \times 10^{10}$ J.

(ii) Calculate the total energy of the satellite.

3. (*a*) (i) State what is meant by *gravitational field strength*.

(ii) The gravitational field strength at the surface of Mars is $3{\cdot}7\ \text{N kg}^{-1}$.

The radius of Mars is $3{\cdot}4 \times 10^3$ km.

(A) Use Newton's universal law of gravitation to show that the mass of Mars is given by the equation

$$M = \frac{gr^2}{G}$$

where the symbols have their usual meaning.

(B) Calculate the mass of Mars.

(*b*) A spacecraft of mass 100 kg is in circular orbit 300 km above the surface of Mars.

(i) Show that the force exerted by Mars on the spacecraft is $3{\cdot}1 \times 10^2$ N.

(ii) Calculate the period of the spacecraft's orbit.

4. (*a*) The Moon orbits the Earth due to the gravitational force between them.

(i) Calculate the magnitude of the gravitational force between the Earth and the Moon.

(ii) Hence calculate the tangential speed of the Moon in its orbit around the Earth.

(iii) Define the term *gravitational potential* at a point in space.

(iv) Calculate the potential energy of the Moon in its orbit.

(v) Hence calculate the total energy of the Moon in its orbit.

(*b*) (i) Derive an expression for the escape velocity from the surface of an astronomical body.

(ii) Calculate the escape velocity from the surface of the Moon.

Total Marks